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DOI: https://doi.org/10.1140/epjc/s10052-016-4274-7



Originally published at:
Bordone, Marzia; Isidori, Gino; Pattori, Andrea (2016). On the standard model predictions for RK and RK∗. European Physical Journal C - Particles and Fields, 76(8):440-446.
DOI: https://doi.org/10.1140/epjc/s10052-016-4274-7

# On the Standard Model predictions for $R_K$ and $R_{K^*}$

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**Abstract** We evaluate the impact of radiative corrections in the ratios $\Gamma[B \to M\mu^+\mu^-]/\Gamma[B \to Me^+e^-]$ when the meson $M$ is a $K$ or a $K^*$. Employing the cuts on $m^2_{\ell\ell}$ and the reconstructed $B$-meson mass presently applied by the LHCb Collaboration, such corrections do not exceed a few %. Moreover, their effect is well described (and corrected for) by existing Montecarlo codes. Our analysis reinforces the interest of these observables as clean probe of physics beyond the Standard Model.

## 1 Introduction

The Lepton Flavor Universality (LFU) ratios

$$R_M[q^2_{\rm min},\, q^2_{\rm max}] = \frac{\displaystyle\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{\mathrm{d}\Gamma(B \to M\mu^+\mu^-)}{\mathrm{d}q^2}}{\displaystyle\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{\mathrm{d}\Gamma(B \to Me^+e^-)}{\mathrm{d}q^2}}\,, \tag{1}$$

where $q^2 = m^2_{\ell\ell}$, are very clean probes of physics beyond the Standard Model (SM): they have small theoretical uncertainties and are sensitive to possible new interactions that couple in a non-universal way to electrons and muons [1]. A strong interest in $R_K$ has recently been raised by the LHCb result [2]

$$R_K\left[1\ \mathrm{GeV}^2,\ 6\ \mathrm{GeV}^2\right] = 0.745^{+0.090}_{-0.074} \pm 0.036\,, \tag{2}$$

that differs from the naïve expectation

$$R^{(\rm SM)}_{K^{(*)}} = 1 \tag{3}$$

by about $2.6\sigma$. The interest is further raised by the combination of this anomaly with other $b \to s\ell^+\ell^-$ observables [3,4], and by the independent hints of violations of LFU observed $B \to D^{(*)}\tau\nu_\ell$ decays [5–7].

While perturbative and non-perturbative QCD contributions cancel in $R_{K^{(*)}}$ (beside trivial kinematical factors), this is not necessarily the case for QED corrections. In particular, QED collinear singularities induce corrections of order $(\alpha/\pi)\log^2(m_B/m_\ell)$ to $b \to s\ell^+\ell^-$ transtions [8,9] that could easily imply 10% effects in $R_{K^{(*)}}$. The purpose of this paper is to estimate these corrections and to precisely quantify up to which level a deviation of $R_K$ or $R_{K^*}$ from 1 can be considered a clean signal of physics beyond the SM.

## 2 QED corrections in $R_M$

A complete evaluation of QED corrections to $B \to M\ell^+\ell^-$ decay amplitudes is a non-trivial task, due to the interplay of perturbative and non-perturbative dynamics (see e.g. [10]). However, the problem is drastically simplified if we are only interested in the LFU ratios $R_M$, especially in the low dilepton invariant mass region, and if interested in possible deviations from Eq. (3) exceeding 1%. In this case the problem is reduced to evaluating $\log(m_\ell)$ enhanced terms, whose origin can be unambiguously traced to soft and collinear photon emission. The latter represents a universal correction factor [11, 12] that can be implemented, by means of appropriate convolution functions,[1] irrespective of the specific short-distance structure of the amplitude.

### 2.1 Universal radiation function

Following the above observation, the treatment of soft and collinear photon emission in $B \to M\ell^+\ell^-$ closely resemble that applied to $h \to 2e2\mu$ decays in Ref. [14]. The key observable we are interested in is the differential lepton-pair

[1]For a discussion about the implementation of universal QED corrections in a general EFT context see also Ref. [13].

invariant-mass distribution

$$\mathscr{F}_M^\ell(q^2) = \frac{d\Gamma(B \to M\ell^+\ell^-)}{dq^2} \; . \tag{4}$$

The complete structure of infrared (IR) divergences in the decay is channel dependent [10]; however, the $\log(m_\ell)$ enhanced terms can be factorized and are independent from the spin of the meson $M$.

The leading QED corrections can be unambiguously identified working in the limit of massless leptons, retaining only the mass terms regulating collinear singularities. In this limit we define the radiator $\omega(x, x_\ell)$, that represents the probability density function that a dilepton system retains a fraction $\sqrt{x}$ of its original invariant mass after bremsstrahlung. Namely we define $x = q^2/q_0^2$, where $q_0^2$ is the initial dilepton invariant mass squared (pre bremsstrahlung), and we introduce the variable $x_\ell = 2m_\ell^2/q_0^2$ that regulates collinear singularities. In order to match the IR-safe observable directly probed in experiments, the integration range of $x$ is determined by the requirement that the reconstructed $B$-meson mass ($m_B^{\rm rec}$), from the measurement of leptons and hadron momenta, is above a minimum value.

In order to regulate IR divergences, we introduce an (unphysical) IR-regulator $x_*$ ($x_* \ll 1$), defined as the minimal detectable value of of $1-x$. The full radiator $\omega(x, x_\ell)$ is then decomposed as

$$\omega(x, x_\ell) = \omega_1(x, x_\ell)\theta(1 - x - x_*) + \omega_2(x, x_\ell, x_*)\delta(1 - x) \; , \tag{5}$$

where the explicit form of $\omega_{1,2}$ in the limit $(1-x) \ll 1$ and $x_\ell, x_* \ll 1$ is

$$\begin{aligned}
\omega_1(x, x_\ell) &= \frac{\alpha}{\pi}\frac{1}{1-x}\left[-2 + (1+x^2)\log\left(\frac{2x}{x_\ell}\right)\right] \; , \\
\omega_2(x, x_\ell, x_*) &= 1 - \frac{\alpha}{\pi}\left\{\frac{5}{4} - \frac{\pi^2}{3} + 2\log(x_*)\right. \\
&\qquad \left. + \left[\frac{3}{2} + 2\log(x_*)\right]\log\left(\frac{x_\ell}{2}\right)\right\} \; .
\end{aligned} \tag{6}$$

The first term, $\omega_1$, describes the real emission of a photon such that the lepton pair retains a fraction $\sqrt{x}$ of its invariant mass; the $\theta$-function implements the corresponding IR regulator. The second term, $\omega_2$, describes the events in which the soft radiation is below the IR regulator, as well as the effect of virtual corrections.

We have determined the structure of $\omega_1$ by means of an explicit $O(\alpha)$ calculation of the real emission, while $\omega_2$ has been determined by the condition

$$\omega_2(x, x_\ell, x_*) = 1 - \int_{2x_\ell}^{1-x^*} dx \; \omega_1(x, x_\ell) \tag{7}$$

that, by construction, ensure the independence of the full radiator from the IR regulator and the normalization condition

$$\int_{2x_\ell}^{1} dx \; \omega(x, x_\ell) = 1 \; . \tag{8}$$

The latter is valid up to finite (non log-enhanced) corrections of $O(\alpha/\pi)$ that define the accuracy of our approximation.

We can thus write the double differential distribution in terms of the invariant mass of the dilepton system before bremsstrahlung and $x = q^2/q_0^2$ as

$$\frac{d^2\Gamma}{dq_0^2 dx} = \mathscr{F}_M^{(0)}(q_0^2)\,\omega(x, x_\ell, x_*) \; , \tag{9}$$

where $\mathscr{F}_M^{(0)}(q_0^2)$ denotes the non-radiative spectrum. Starting from Eq. (9) we can extract the double differential spectrum after radiative corrections. To this purpose, we first trade $x$ for $q^2$, we then integrate over all the possible values of $q_0^2$ determined by the cut on $m_B^{\rm rec}$, namely[2]

$$q_0^2 \leq q_{0,\max}^2(q^2, \delta) = \frac{q^2}{\delta^2}\left[1 + (1-\delta^2)\frac{m_M^2}{m_B^2\delta^2 - q^2}\right] \; , \tag{10}$$

where $\delta = m_B^{\rm rec}/m_B < 1$. Proceeding this way we finally obtain:

$$\mathscr{F}_M^\ell(q^2) = \int_{q^2}^{q_{0,\max}^2} \frac{dq_0^2}{q_0^2}\mathscr{F}_M^{(0)}(q_0^2)\,\omega\left(\frac{q^2}{q_0^2}, \frac{2m_\ell^2}{q_0^2}\right) , \tag{11}$$

We stress that the result in Eq. (11) includes both real and virtual QED corrections. The latter have been indirectly determined by the normalization condition for $\omega(x, x_\ell)$, that is the same condition applied in showering algorithms [15], and that follows from the safe IR behavior of the photon-inclusive dilepton spectrum.

Before concluding this section, we summarize below the size of neglected contributions and the accuracy of this calculation.

- As anticipated, we do not control $O(\alpha/\pi)$ virtual corrections that are regular in the limit $m_\ell \to 0$. The latter are expected to be safely below the 1% level.

[2]In principle, from a pure kinematical point of view, the cut on $m_B^{\rm rec}$ allow $q_0^2$ values even exceeding the bound in Eq. (10); however, this occurs only for non-soft and non-collinear emissions that are beyond our approximations.

- The calculation of the real emission has been done in the limit $m_\ell^2 \ll q^2$ that is certainly an excellent approximation in the electron case, while it is less good in the muon case; however, also in this case the neglected contributions are $O(\alpha/\pi)$ non log-enhanced terms.
- In the case of a charged meson in the final state, we should consider also the radiation from the meson leg. We have checked by means of an explicit calculation at $O(\alpha)$ (employing a generic hadronic matrix element) that the latter do not interfere with the radiation of the lepton legs at the leading-log level once we integrate over the leptonic angles.[3] The radiation of the meson leg can thus be considered separately by means of an independent radiation function. A quantification of its effect in the $B^+ \to K^+\ell^+\ell^-$ case is discussed in sect. 3.
- Independently of the charge of the meson, an additional contribution to the real radiation is due to structure-dependent terms (i.e. separately gauge-invariant amplitudes that vanish in the $E_\gamma \to 0$ limit). By construction, these amplitudes are free from soft singularities but could have collinear singularities. However, these vanishes after a symmetry integration over the leptonic angles for the same argument discussed above.
- In order to quantify the impact of radiative corrections we need a theoretical input for the non-radiative spectrum $\mathscr{F}_M^{(0)}(q_0^2)$, whose explicit expression for $B \to K$ and $B \to K^*$ transitions is discussed in sect. 2.2. From Eq. (11) it is clear that, as long as $\mathscr{F}_M^\ell(q^2)/\mathscr{F}_M^{(0)}(q^2)$ is a smooth function of $q^2$, the relative impact of radiative corrections in $R_M$ is insensitive to the dynamics responsible for the $B \to M\ell^+\ell^-$ decay.

### 2.2 Parameterization of the non-radiative spectrum

The choice of the radiative spectrum for the $B \to K^+\ell^+\ell^-$ decay is quite simple. In full generality we can write

$$\mathscr{F}_K^{(0)}(q^2) \propto \lambda^{3/2}(q^2)\left|f_+(q^2)\right|^2\left[|a_9(q^2)|^2 + |a_{10}|^2\right] , \qquad (12)$$

where $\lambda(s) = (m_B^4 + m_K^4 + s^2 - 2m_K^2 m_B^2 - 2sm_B^2 - 2sm_K^2)/m_B^4$, $f_+(q^2)$ is the $B \to K$ vector form factor

$$\langle K(k)|\bar{s}\gamma_\mu b|B(p)\rangle = f_+(q^2)(p+k)^\mu + O(q^\mu) \qquad (13)$$

and $a_9(q_0^2)$ and $a_{10}$ denote the effective Wilson coefficients of the vector and the axial-vector components of the leptonic current [16]. For our numerical analysis we use the parameterization of the form factor and the numerical values of the Wilson coefficients from Ref. [16].

In order to provide an effective description of the non-perturbative distortion of the spectrum induced by the charmonium resonances, we modify the vector effective Wilson coefficient as follows

$$a_9(q^2) = a_9^{\rm pert}(q^2) + \kappa_\psi \frac{q^2}{q^2 - m_\psi^2 + i m_\psi \Gamma_\psi} \qquad (14)$$

where $\{m_\psi, \Gamma_\psi\}$ are the experimental mass and width of the $J/\psi(1S)$ state, and the value of the (real) effective coupling $\kappa_\psi$ has been fixed in order to reproduce $\mathscr{B}(B \to K\psi)$ in the narrow width approximation. This description is certainly approximate (see e.g. the discussion in Ref. [17, 18]), but it provides a good estimate of the region where the $B \to K^+\ell^+\ell^-$ spectrum starts to vary rapidly with $q^2$, that is relevant in order to define the region of validity of our approach.

As far as the $B \to K^*\ell^+\ell^-$ is concerned, we proceed introducing the standard set of vector, axial, and tensor form factors

$$\langle K^*|\bar{s}\gamma_\mu b|B\rangle = \frac{2V(q^2)}{m_B + m_V}\varepsilon_{\mu\rho\sigma\tau}\varepsilon^{*\rho}p^\sigma k^\tau, \qquad (15)$$

$$\begin{aligned}\langle K^*|\bar{s}\gamma_\mu\gamma_5 b|B\rangle &= i\varepsilon^{*\rho}\left[2m_V A_0(q^2)\frac{q_\mu q_\rho}{q^2}\right.\\ &+(m_B+m_V)A_1(q^2)\left(g_{\mu\rho} - \frac{q_\mu q_\rho}{q^2}\right)\\ &\left.- A_2(q^2)\frac{q_\rho}{m_B+m_V}\left((p+k)_\mu - \frac{\Delta m^2}{q^2}q_\mu\right)\right], \qquad (16)\end{aligned}$$

$$\langle K^*|\bar{s}i\sigma_{\mu\nu}q^\nu b|B\rangle = -2T_1(q^2)\varepsilon_{\mu\rho\sigma\tau}\varepsilon^{*\rho}p^\sigma k^\tau, \qquad (17)$$

$$\begin{aligned}\langle K^*|\bar{s}i\sigma_{\mu\nu}\gamma_5 q^\nu b|B\rangle &= iT_2(q^2)\left[\varepsilon_\mu^*\Delta m^2 - (\varepsilon^*\cdot q)(p+k)_\mu\right]\\ &+iT_3(q^2)(\varepsilon^*\cdot q)\left(q_\mu - \frac{q^2}{\Delta m^2}(p+k)_\mu\right), \qquad (18)\end{aligned}$$

where $\Delta m^2 = m_B^2 - m_{K^*}^2$, whose numerical values are taken from Ref. [19] (and based on the original works in Ref. [20]). With these we proceed evaluating the differential rate as, for instance, in Ref. [1].

## 3 Numerical results

The relative impact of radiative corrections in $B \to K^+\ell^+\ell^-$, namely a plot of the ratio

$$\mathscr{R}_K^\ell(q^2) = \frac{\mathscr{F}_K^\ell(q^2)}{\mathscr{F}_K^{(0)}(q^2)}, \qquad (19)$$

[3] This happens because the leptonic current carries an overall neutral electric charge.

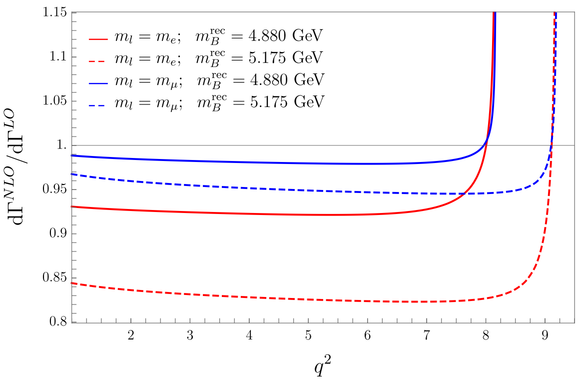


**Fig. 1** Relative impact of radiative correction in $B \to K^+\ell^+\ell^-$ decays for $q^2 \in [1, 9.5]$ GeV$^2$, with different cuts on the reconstructed mass and different lepton masses.

| $B \to K\ell^+\ell^-$ | $\ell = e$ | $\ell = \mu$ |
|---|---|---|
| $m_B^{rec} = 4.880$ GeV | $-7.6\%$ | $-1.8\%$ |
| $m_B^{rec} = 5.175$ GeV | $-16.9\%$ | $-4.6\%$ |

| $B \to K^*\ell^+\ell^-$ | $\ell = e$ | $\ell = \mu$ |
|---|---|---|
| $m_B^{rec} = 4.880$ GeV | $-7.3\%$ | $-1.7\%$ |
| $m_B^{rec} = 5.175$ GeV | $-16.7\%$ | $-4.5\%$ |

**Table 1** Relative impact of radiative corrections for $q^2 \in [1, 6]$ GeV$^2$, with different cuts on the reconstructed mass and different lepton masses.

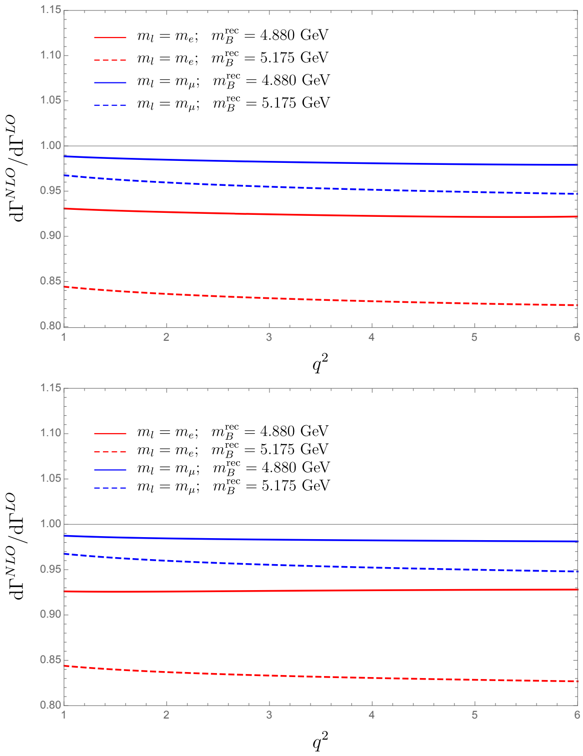


**Fig. 2** Relative impact of radiative correction in $B \to K\ell^+\ell^-$ (up) and in $B \to K^*\ell^+\ell^-$ (down) for $q^2 \in [1, 6]$ GeV$^2$, with different cuts on the reconstructed mass and different lepton masses. .

is shown in Fig. 1 in the region $q^2 \in [1, 9]$ GeV$^2$. The different colors correspond to different lepton masses (red for the electron and blue for the muon). Dashed and full lines correspond to different choices of the minimal cut on the reconstructed $B$-meson mass from the momenta of charged particles. We have choosen for the latter the two values used in Ref. [2] for the analysis of the electron modes ($m_B^{rec} \geq 4.880$ GeV, full lines) or the muon modes ($m_B^{rec} \geq 5.175$ GeV, dashed lines).

The first point to be noted in Fig. 1 is that $\mathscr{R}_K^\ell(q^2)$ is a smooth function for sufficiently low values of $q^2$, while a sudden rise appear close to the resonance region. The latter is a manifestation of the radiative return from the $J/\Psi$ peak. The position where the $J/\Psi$ contamination appears depends only from the cut imposed on $m_B^{rec}$. Even for the looser cut applied in the electron case the region $q^2 \in [1, 6]$ GeV$^2$ is free from the $J/\Psi$ contamination and can be estimated with good theoretical accuracy (see Fig. 2). To better quantify this statement we have explicitly checked that varying the phase of the effective coupling $\kappa_\psi$ in Eq. (14) leads to per-mill modifications to $\mathscr{R}_K^\ell(q^2)$ for $q^2 \leq 6$ GeV$^2$. We also have explicitly checked that the cut on $m_B^{rec}$ eliminates photons from the $J/\Psi$ peak also when considering the full kinematics of the event, i.e. beyond the soft and collinear approximation on which we derived Eq. (10).

The second point to be noted is that in the regular region of the spectrum radiative corrections reach (or even exceed) the 10% level for the electrons (as naively expected); however, the net effect in $R_K$ is significantly smaller. Indeed the magnitude of the corrections is larger for electron vs. muons, but it increases for $m_B^{rec} \to m_B$. This imply that the specific choice of $m_B^{rec}$ cuts applied by the LHCb collaboration, i.e. a loose cut for the electrons and a tighter cut for the muons, give rise to a natural compensation of the QED corrections to $R_K$.

The integrated corrections that quantity the modifications to $R_K$ are reported in Table 1. Given the choice of $m_B^{rec}$ applied in Ref. [2], we estimate that radiative corrections induce a *positive* shift of the central value of $R_K$ of a about $\Delta R_K = +3\%$. This effect is taken into account by the LHCb collaboration, who estimated the impact of radiative correc-

| | |
|---|---|
| $m_B^{\rm rec} = 4.880$ GeV | $-0.02\%$ |
| $m_B^{\rm rec} = 5.175$ GeV | $-0.18\%$ |

**Table 2** Relative contribution of radiative corrections due emission from the meson leg, in the $B^+ \to K^+\ell^+\ell^-$ case, for $q^2 \in [1,6]$ GeV$^2$.

tions with PHOTOS [15], and properly corrected for in the result reported. We have explicitly checked that our estimate of $\Delta R_K$ is in agreement with that obtained with PHOTOS up to differences within $\pm 1\%$.[4]

In order to check the smallness of the non-log$(m_\ell)$ enhanced terms, in Table 2 we report the effect of the radiation from the meson leg, that is IR divergent but has no collinear singularities. We evaluated these terms developing the corresponding radiator function (see Ref. [13]), whose implementation depend only on $m_B^{\rm rec}$. As can be seen from Table 2, the results are well below the 1% level.

The impact of radiative corrections in the $B \to K^*\ell^+\ell^-$ decays is shown in Fig. 2 and summarized by the integrated values reported in Table 1. The situation is very similar to the $B^+ \to K^+\ell^+\ell^-$ : employing the same $m_B^{\rm rec}$ cuts for electron and muon modes as in Ref. [2], we find that the net impact of radiative corrections is $\Delta R_{K^*} = +2.8\%$. Also in this case this effect is well described by PHOTOS and therefore can be properly corrected for in future experimental analyses.

## 4 Conclusions

The experimental result in Eq. (2) ha stimulated a lot of theoretical activity [21–49] In view of this result and, especially, in view of possible future experimental improvements in the determination of $R_K$ or $R_{K^*}$, we have re-examined the SM predictions of these LFU ratios.

As we have show, log$(m_\ell)$-enhanced QED corrections may induce sizable deviations from Eq. (3), even up to 10%, depending on the specific cuts applied to define physical observables. In particular, a key role is played by the cuts on $q^2 = m_{\ell\ell}^2$ and on the reconstructed $B$-meson mass. The former is important to avoid rapidly varying regions in the dilepton spectrum (where the theoretical tools to compute QED corrections become unreliable), while the latter defines the physical IR cut-off of the rates. Employing the cuts presently applied by the LHCb Collaboration, the corrections in $R_K$ do not exceed 3%. Moreover, their effect is well described (and corrected for in the experimental analysis) by existing Montecarlo codes.

According to our analysis, a deviation of $R_K$ or $R_{K^*}$ from 1 exceeding the 1% level, performed along the lines of Ref. [2] in the region 1 GeV$^2 < q^2 < 6$ GeV$^2$, would be a clear signal of physics beyond the Standard Model.

[4] We thank Rafael Silva Coutinho for a detailed comparison about the radiative corrections implemented in the LHCb analysis of $R_K$.

**Acknowledgements** We thank Rafael Silva Coutinho and Nicola Serra for useful discussions about the LHCb analysis of $R_K$, and Danny van Dyk for help with the numerical implementation of $B \to K^*$ form factors. This research was supported in part by the Swiss National Science Foundation (SNF) under contract 200021-159720.

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